Welcome to Algebra I! Algebra at its core is all about using the properties of numbers (how they behave) to manipulate unknowns, called variables. But, in practicality, Algebra is used to recognize patterns, turn them into mathematical relationships, and then use these relationships for useful purposes. Today’s lesson, being the first of the course, is exploratory in nature and will utilize a basic understanding of rates or ratios.

Exercise #1: Answer the following rate/ratio questions using multiplication and division. Show your calculation (and keep track of your units!).

(a) If there are 12 eggs per carton, then how many eggs do we have in 5 cartons?

\[
x \text{ eggs} = 12 \text{ eggs} \times 5 \text{ cartons} = 60 \text{ eggs}
\]

(b) If a car is traveling at 65 miles per hour, then how far does it travel in 2 hours?

\[
\frac{65 \text{ miles}}{1 \text{ hour}} = \frac{x \text{ miles}}{2 \text{ hours}}
\]

\[
x = 65 \times 2 = 130 \text{ miles}
\]

(c) If a pizza contains 8 slices and there are 4 people eating, how many slices are there per person?

\[
\frac{8 \text{ slices}}{4 \text{ people}} = 2 \text{ slices per person}
\]

(d) If a hiker travels 20 miles in one hour, how many minutes does it take per mile traveled?

\[
\frac{60 \text{ mins}}{20 \text{ miles}} = 3 \text{ mins per mile}
\]

60 mins = 1 hour
Rates show up everywhere in the real world, whether it is your pay per hour of work or the texts you can send per month. Rates are all about multiplication and division because they ultimately are a ratio of two quantities, both of which are changing or varying.

Exercise #2: A runner is traveling at a constant rate of 8 meters per second. How long does it take for the runner to travel 100 meters?

(a) Experiment solving this problem by setting up a table to track how far the runner has moved after each second.

<table>
<thead>
<tr>
<th>time, $t$ (seconds)</th>
<th>Distance, $D$ (meters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>40</td>
</tr>
<tr>
<td>10</td>
<td>80</td>
</tr>
</tbody>
</table>

(b) Create an equation that gives the distance, $D$, that the person has run if you know the amount of time, $t$, they have been running.

$$D = 8t$$

(multiply time by 8)

(c) Now, set up and solve a simple algebraic equation based on (b), that gives the exact amount of time it takes for the runner to travel 100 meters.

$$\frac{D}{8} = \frac{80}{8}$$

$$12.5 = t$$ sec
The previous exercise showed how we can take a pattern and extend it into the world of algebra, a world that contains symbols and conventions that may seem strange, but hopefully somewhat familiar from previous work. In the final exercise, we will tackle a larger problem to see how rates, patterns, and algebra can combine to solve a more challenging problem.

Exercise #3: A man is walking across a 300 foot long field at the same time his daughter is walking towards him from the opposite end. The man is walking at 9 feet per second and the daughter is moving at 6 feet per second. How many seconds will it take them to meet somewhere in the middle?

(a) Draw a diagram to help keep track of where the man and his daughter are after 1 second, 2 seconds, 3 seconds, etcetera. Create a table as well that helps keep track of how far each one of them has traveled as time goes on.

<table>
<thead>
<tr>
<th>Time (seconds)</th>
<th>Father's Distance (feet)</th>
<th>Daughter's Distance (feet)</th>
<th>Total Distance (feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9</td>
<td>6</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
<td>12</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>45</td>
<td>30</td>
<td>75</td>
</tr>
<tr>
<td>10</td>
<td>90</td>
<td>60</td>
<td>150</td>
</tr>
</tbody>
</table>

150 \cdot 2 = 300 \text{ ft}

10 - 2 = 8 \text{ secs}
(b) What must be true about the distances the two have traveled when they meet somewhere in the middle?

The sum must be 300.

(c) Create equations similar to Exercise #3 to predict the distance the father has traveled and the distance the daughter has traveled.

\[
\begin{align*}
\text{Father} & : D = 9t \\
\text{Daughter} & : D = 6t
\end{align*}
\]

(d) Create and solve an equation to predict the exact amount of time it takes for the father and daughter to meet in the middle.

\[
9t + 6t = 300 \\
15t = 300 \\
\frac{15t}{15} = \frac{300}{15} \\
t = 20 \text{ sec}
\]