Lesson 4: Dilations Mapping Segments, Lines, Rays and Circles

Opening Exercise

Segment $\overline{PQ}$ has endpoints $P(-3, 1)$ and $Q(3, 2)$. Graph segment $\overline{PQ}$ on the axes below and graph its image after a dilation from the origin with a scale factor of 3.

\[ \frac{m_{P'Q'}}{m_{PQ}} = \frac{3}{18} = \frac{1}{6} \]
\[ \frac{m_{PQ}}{m_{PQ}} = \frac{1}{6} \]

$\overline{P'Q'} \parallel \overline{PQ}$

$3(\overline{PQ}) = \overline{P'Q'}$

Is a dilated segment still a segment?
Yes!

What is the relationship between $\overline{PQ}$ and its image after the dilation? (Two things!)
Example 1

You will need a compass and a straightedge

Dilate $PQ$ by a scale factor of 2 from center $O$.

Is the image still a segment?

Duh! Sure.

What is the relationship between $PQ$ and its image after the dilation?

$P'Q' \parallel PQ$

$P'Q' = 2PQ$

$\frac{P'Q'}{PQ} = \frac{2}{1}$
Example 2

You will need a compass and a straightedge

Dilate \( \overline{PQ} \) by a scale factor of 2 from center \( O \).

\[ \overrightarrow{PQ} \parallel \overrightarrow{P'Q} \]

Is the image still a line?

Yes

What is the relationship between \( \overline{PQ} \) and its image after the dilation?

\[ \overrightarrow{PQ} \parallel \overrightarrow{P'Q} \]

Dilated line is not proportional to original because they both have infinite length.
Example 3

You will need a compass

Dilate $\overline{PQ}$ by a scale factor of 3 from center $R$.

How much longer is $\overline{P'Q'}$ than $\overline{PQ}$?

$P'Q' = 3 \overline{PQ}$

What do you notice about line $\overline{PQ}$ vs. line $\overline{P'Q'}$?

$\overline{PQ}$ and $\overline{P'Q'}$ are the same line.
Summary

- The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
- A dilation takes a line not passing through the center of dilation to a parallel line.
- A dilation of a line passing through the center of dilation leaves the line unchanged.
Example 4

You will need a compass and a straightedge.

Dilate circle $O$ by a scale factor of 2 with $O$ being the center of dilation.

$C_1 = \pi d$

$C_2 = \pi 2d = 2\pi d$

What do you notice about the centers of the circle given and its image after the dilation?

Same center

What is true about the radii?

The radius of the image circle is twice the radius of the original circle.
**Vocabulary**

<table>
<thead>
<tr>
<th>Define</th>
<th>Diagram</th>
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<tbody>
<tr>
<td>Concentric Circles: two circles that have the same center but different radii.</td>
<td><img src="image" alt="Diagram" /></td>
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Example 5

You will need a compass and a straightedge

Dilate circle $A$ by a scale factor of 2 with $O$ being the center of dilation.

What is the relationship between circle $A$'s radius and the radius of its image after the dilation?

Radius of image, circle $O$ is 4 times the radius of circle $A$. 
Example 6

Find the center of dilation that would map circle $O$ to circle $O'$.

Use your center of dilation to locate point $W$ on circle $O$. 
Homework

You will need a compass and a straightedge

Dilate $\overline{AB}$ by a scale factor of 4 from center $O$.

Is the image still a ray?

What is the relationship between ray $\overrightarrow{AB}$ and its image after the dilation?

$\overrightarrow{AB} \parallel \overrightarrow{A'B'}$

What is the relationship between segment $AB$ and segment $A'B'$?

$\overline{AB} \parallel \overline{A'B'} \quad A'B' = 4AB \quad \frac{AB}{A'B'} = \frac{1}{4}$

$\frac{A'B'}{AB} = \frac{4}{1}$
2. Dilate circle \( O \) by a scale factor of 3 with \( O \) being the center of dilation. What kind of circles are formed?

\[
\frac{OA'}{OA} = \frac{3}{1}
\]

\( OA' = 3 \)